

Lec 22:

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Accretion Columns:

If the compact object at the center is strongly magnetized, the disk will not extend all the way down to the stellar surface. The radial infall will be disrupted some distance away from the star in this case. This is intuitively expected as the magnetic field tends to whirl the particles around.

The magnetic field may be quite complex near the surface of the compact object. However, all of the multipole moments higher than the dipole fall off very rapidly away from the star. It is therefore sufficient to keep the contribution from the magnetic dipole moment \vec{m} , which results in:

$$\vec{B} = \frac{3\hat{n}(\hat{n} \cdot \vec{m}) - \vec{m}}{r^3}$$

where the disk is located,

On the horizontal plane, we have:

$$B(R) = B_* \left(\frac{R_*}{R} \right)^3$$

Here B_* is the magnetic field at the polar cap ($R=R_*$), and R denotes radius in the equatorial plane.

A simple criterion for the dominance of the magnetic field is that the pressure in the magnetic field becomes larger than the matter pressure (basically the ram pressure):

$$\frac{1}{8\pi} B^2 \gtrsim \frac{GM\dot{M}}{R}$$

From previous lectures, we have:

$$\dot{M} \sim \frac{M}{12\pi} \frac{1}{\xi \alpha} \left(\frac{v}{R_g T} \right) \left(\frac{GM}{R^3} \right)^{1/2}$$

The magnetic radius R_m is then found to be:

$$R_m \approx B_* R_*^3 \left(\frac{R_g T}{v} \right)^{3/4} \frac{1}{GM} \left(\frac{3}{4} \alpha \right)^{1/2} \dot{M}^{-1/2}$$

It can be written in a useful form,

$$R_m \approx (3.0 \text{ km}) \alpha^{1/2} \left(\frac{B_*}{10^{12} \text{ G}} \right) \left(\frac{R_*}{10^6 \text{ cm}} \right)^3 \left(\frac{T}{10^6 \text{ K}} \right)^{3/4} \left(\frac{M}{M} \right) \left(\frac{\dot{M}}{10^{16} \text{ g s}^{-1}} \right)^{-1/2}$$

For a neutron star, when parameters take their

natural values used in the above normalization, we find $R_m \sim 3 R_*$. This suggests that the radial flow of material and the disk are disrupted relatively far away from the surface of the star.

Once the gas reaches the magnetic radius, $R \sim R_m$, it is funneled toward the polar caps of the star along the magnetic field lines. This is the only possibility for motion when the magnetic field is very strong. This cropping of the disk is indeed seen in both systems where the compact object is a magnetized white dwarf (Cataclysmic Variable) or a neutron star (pulsar). The physical conditions in the latter are much more extreme ($B_* \sim 10^{12}$ G, and gravitational free-fall velocities $\sim \frac{c}{2}$ at R_*), and hence more difficult to model. Magnetic Cataclysmic variables, in which the

primary is a magnetized white dwarf, are better understood than X-ray pulsars (their neutron star counterparts).

Accretion Columns in Magnetic Cataclysmic Variables;

In these systems the primary is a magnetized white dwarf. As mentioned, it is sufficient to keep the magnetic dipole term at sufficiently large distances from the star's center. A dipole field line is described by the following relation:

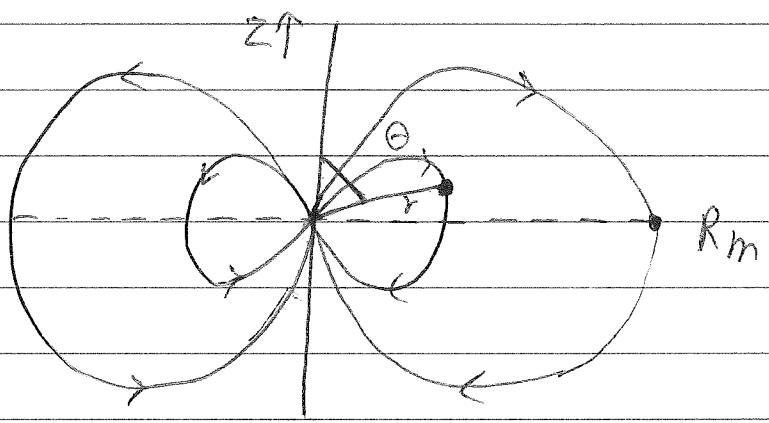
line is described by the following relation:

$$r = \text{Const.} \times \sin^2 \theta$$

$$\theta = \frac{\pi}{2} \Rightarrow r = R_m$$

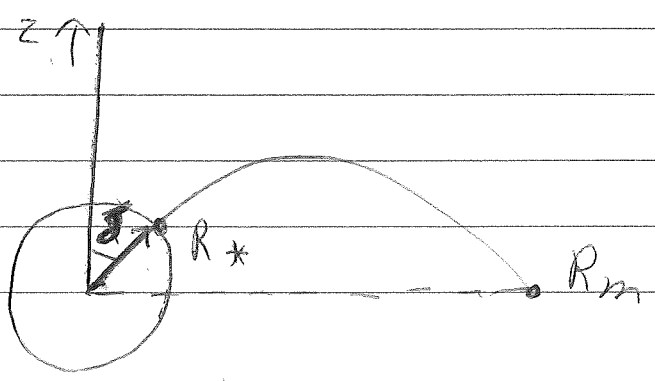
Thus:

$$\text{Const.} = R_m \Rightarrow r = R_m \sin^2 \theta$$



The outer angle of impact at the white dwarf's surface is,

$$\delta = \sin^{-1} \left(\frac{R_*}{R_m} \right)^{\frac{1}{2}}$$



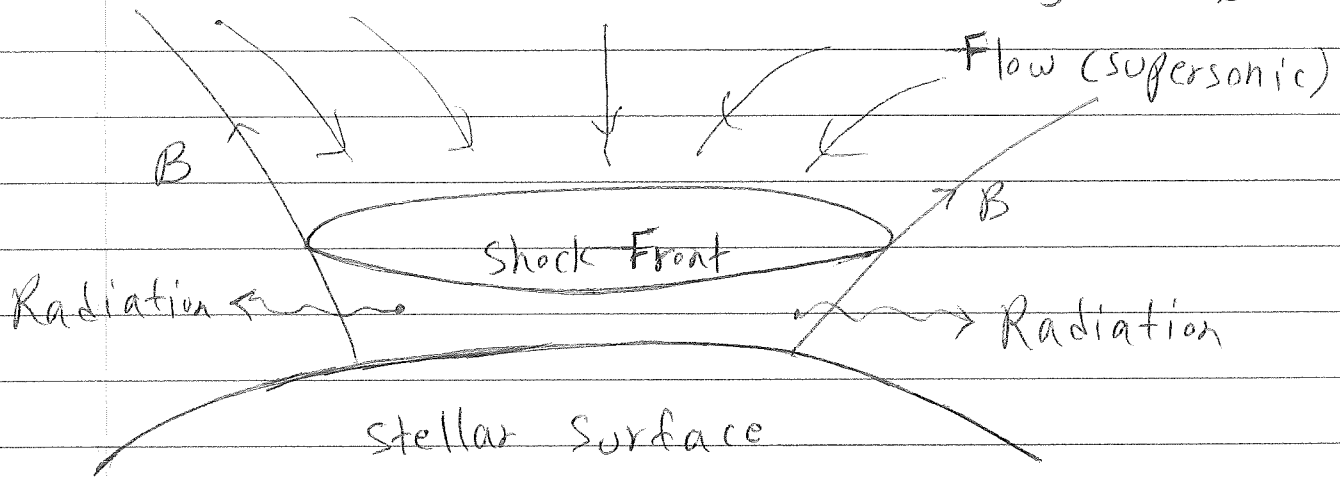
For a white dwarf:

$$R_* \sim 10^9 \text{ cm}, \quad B_* \sim 2 \times 10^7 \text{ G}, \quad T \sim 10^5 \text{ K}$$

The latter result in:

$$R_m \sim 10^{10} \text{ cm}$$

Hence $\delta \sim 20^\circ$ for a white dwarf primary. We note that the magnetic field lines thread the disk beyond R_m and feed the termination funnel at smaller δ . This makes the entire region $\theta \leq \delta$ full.



For $\delta \sim 20^\circ$, the polar cap area beneath the flow is $\sim \frac{1}{40}$ of the total stellar surface area. Since both polar caps can be active, a fraction $f \sim \frac{1}{20}$ of the entire white dwarf's

surface participates in the accretion.

Inside the funnel, the falling matter reaches the free-fall velocity,

$$v_{ff} = \left(\frac{2GM}{R_*} \right)^{1/2} \sim 5 \times 10^8 \text{ cm s}^{-1}$$

The thermal velocity of electrons at temperatures relevant

for a white dwarf is $v_{th} \sim \left(\frac{kT}{m_e} \right)^{1/2} \sim 10^8 \text{ cm s}^{-1}$ (and much smaller

than this for protons). This is roughly the speed of sound in

the medium, which implies that the inflow is supersonic. As a

result, it produces a shock that heats up the material in the

region behind the shock.

We recall that across a strong shock the relative velocity of

the gas drops by a factor of 4. Conservation of the mass

then implies that the density should increase by the same

factor. The density in the shocked region will therefore be,

$$\rho_{shock} \approx 4 \frac{\dot{M}}{4\pi R_*^2} \left(\frac{2GM}{R_*} \right)^{-1/2} \sim 10^{10} \text{ g cm}^{-3}$$

The corresponding temperature is:

$$T_{\text{shock}} \sim \frac{m_H}{3k} \left(\frac{3v_{\text{ff}}}{4} \right)^2 \sim 6 \times 10^8 \text{ K}$$

The plasma is thus heated up by the shock quite considerably.

It then cools off quickly as photons leave the system without being trapped. This can be seen by finding the optical depth

in the transverse direction:

$$\tau_{\perp} \sim \left(\frac{\rho_{\text{shock}}}{\mu m_H} \right) \sigma_T f^{\frac{1}{2}} R_* \sim 0.03$$

This confirms that the medium is optically thin.

The overall spectrum from accretion columns of a white dwarf

has various components. The dominant one is Bremsstrahlung

radiation in the hard X-ray. There is also cyclotron emission

in the UV (since there is a magnetic field), as well as a soft

blackbody X-ray radiation that comes from reprocessing of

hard X-rays that penetrate below the stellar surface.

Accretion Columns in X-ray Pulsars;

The main complication arising in the case of X-ray pulsars is the larger radiation pressure in this case. Since the accreted plasma is funneled onto smaller polar cap regions, "f" will be much smaller (at least by an order of magnitude) in this case. The radiative flux will therefore be larger than an isotropically emitting source by a factor of $\frac{1}{f}$. This implies that ^{even} with an accretion rate $L_{acc} \sim 0.1 L_{edd}$, the effective luminosity out of the funnel can be comparable to L_{edd} . Once this happens, radiation process significantly affects the accretion.

A second complication is that the transverse optical depth τ_{\perp} is much larger in the case of X-ray pulsars. Since $\tau_{\perp} \propto n_e R_*$, while $n_e \propto R_*^{-2}$, then $\tau_{\perp} \propto R_*^{-1}$. Hence τ_{\perp} in the case of a

neutron star is larger than that for a white dwarf by a factor of $\sim 10^3$, which results in $\tau_{\text{I}} \sim 30$, and hence an optically thick medium.

With an Eddington flux being produced within the funnel, and the medium being optically thick, one may expect to have instabilities. Numerical simulations confirm the view that the accretion column in these circumstances are in fact going to be unstable.

Finally, not only the physical modeling is more involved in the case of X-ray pulsars, but also there is less observational evidence available for making comparisons with theoretical models.